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Low–Energy Kaon–Nucleon Interactions and Scattering at DAΦNE[†]

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Abstract.

We present in this contribution the basic formulæ for the analysis of low–momentum charged– and neutral–kaon interactions in hydrogen, including as well a (brief) description of the problems left open by past experiments, and of the improvements DAΦNE can be expected to offer over them. Interactions in deuterium and other light nuclei will be only briefly mentioned, and only in those respects touching directly upon the more "elementary" aspects of kaon–nucleon interactions.

1. Introduction.

DAΦNE is expected to produce, with "standard" assumptions about luminosity and cross sections, about 1.25×10^{10} $K^{\pm}$'s and 8.5×10^{9} K^0_L's per year of operation, considering a conventional "Snowmass year" of 10^7 s. With a detector of KLOE's size one can thus expect to observe up to millions of interactions per year, even in the low–density gas (mostly helium at atmospheric pressure) filling its fiducial volume.

A first question has thus to be answered: do these events contain useful physics to be worth recording and interpreting? One could even go further and envisage using DAΦNE as a source of high–resolution ($\Delta p/p$ of respectively 1.1×10^{-2} for $K^{\pm}$'s and 1.5×10^{-2} for K^0_L's), low–momentum kaons (127 MeV/c for the former, minus the losses in the matter along the kaon path from DAΦNE's beam–beam interaction region to the point where

it interacts in the detector, and 110 MeV/c for the latter), to measure with a *dedicated* detector KN and $\overline{K}N$ interactions, e.g. in a toroidal volume filled with gaseous H_2 or D_2, and possibly also with heavier–element, gaseous (i.e. low–density) targets.

The machine has thus the ability to explore a small kinematical region ($90\ MeV/c \leq p_{lab} \leq 120\ MeV/c$), very little investigated in the past: only few bubble–chamber $K^{\pm}$ experiments in hydrogen (and deuterium)[1,2] plus very few data points on K^0_S regeneration[3] exist, all with extremely low statistics and more than a decade old (the last experiment[2] to cover this region was carried out in the second half of the seventies by the TST Collaboration at the hydrogen bubble chamber in NIMROD's low–energy kaon beam). Since in dispersive calculations of low–energy parameters for KN interactions ($\overline{K}NY$ and $\pi YY'$ coupling constants, scattering lengths, σ–terms) the bulk of the uncertainties comes from the integrals over the unphysical regions, for whose description one must extrapolate data analyses for quite an energy range, down from the physical region well above the charge–exchange threshold, DAΦNE, as the cleanest, lowest–energy kaon *source* ever built, can be expected to make substantial improvements over our present knowledge of those parameters.

The following sections are therefore dedicated to illustrating the details (and the limitations) of present–day information on low–energy kaon–nucleon physics, spotlighting those points which still await being clarified, and where DAΦNE can be expected to improve. Being the phenomenology in this case more complex than in the (strictly related) pion–nucleon one, we shall start almost from scratch.

We shall also take the liberty of not going into the details of models, in particular for the spectroscopic classification of the $J^P = \frac{1}{2}^-$, S–wave resonance $\Lambda(1405)$: data are still so scarce, after more than two decades of studies, that any interpretation of such a state is to be regarded as purely conjectural[4,5].

2. Amplitude formalism for two–body KN and $\overline{K}N$ interactions.

Any $a_1(0^-, q) + B_1(\frac{1}{2}^+, p) \rightarrow a_2(0^-, q') + B_2(\frac{1}{2}^+, p')$ process is most economically described in the centre–of–mass (c.m.) frame by two amplitudes, $G(w, \theta)$ and $H(w, \theta)$, when the T–matrix element $T_{\alpha\beta}$ is expressed in terms of the two–component Pauli spinors χ_α and χ_β (respectively for the final and initial $\frac{1}{2}^+$ baryons) as $T_{\alpha\beta} = \chi^\dagger_\alpha \mathbf{T} \chi_\beta$, where

$$\mathbf{T} = G(w, \theta) \times \mathbf{I} + iH(w, \theta) \times (\vec{\sigma} \cdot \hat{n}) \tag{1}$$

and $\hat{n}$ defines the normal to the scattering plane[5].

These c.m. amplitudes have a simple expansion in the partial waves $T_{\ell\pm}(w) = (\eta_{\ell\pm}e^{2i\delta_{\ell\pm}} - 1)/2iq$, given by

$$G_N(w,\theta) = \sum_{\ell=0}^{\infty}[(\ell+1)T_{\ell+}(w) + \ell\; T_{\ell-}(w)]P_\ell(\cos\theta) \tag{2}$$

$$H_N(w,\theta) = \sum_{\ell=1}^{\infty}[T_{\ell+}(w) - T_{\ell-}(w)]P'_\ell(\cos\theta)\ , \tag{3}$$

where the subscript N indicates that only the *purely nuclear* part of the interaction has been considered.

To describe adequately the data, the amplitudes must also include electromagnetism and can be rewritten as

$$G(w,\theta) = \tilde{G}_N(w,\theta) + G_C(w,\theta) \tag{4}$$

$$H(w,\theta) = \tilde{H}_N(w,\theta) + H_C(w,\theta)\ , \tag{5}$$

where the tilded nuclear amplitudes differ from the untilded ones only in the (generally complex) Coulomb shifts $\sigma_{\ell\pm}^{\mathrm{in(fin)}}$ having been applied to each partial wave $T_{\ell\pm}$, namely when

$$T_{\ell\pm}\ \rightarrow\ \tilde{T}_{\ell\pm} = e^{i\sigma_{\ell\pm}^{\mathrm{in}}}T_{\ell\pm}(w)e^{i\sigma_{\ell\pm}^{\mathrm{fin}}}\ . \tag{6}$$

The one–photon–exchange amplitudes G_C and H_C (of course absent for charge– and/or strangeness–exchange processes, but present at $t \neq 0$ for K_S regeneration, which can also go via one–photon exchange) can be expressed in terms of the Dirac nucleon form factors as[6,7] (M and m indicate respectively the baryon and meson masses in the inital state: for final states all quantities will be primed)

$$G_C(w,\theta) = \pm e^{\pm i\phi_C}\cdot\{(\frac{2q\gamma}{t}+\frac{\alpha}{2w}\frac{w+M}{E+M})\cdot F_1(t)+[w-M+\frac{t}{4(E+M)}]\cdot\frac{\alpha F_2(t)}{2wM}\}\cdot F_K(t) \tag{7}$$

and

$$H_C(w,\theta) = \pm\frac{\alpha F_K(t)}{2w\tan\frac{1}{2}\theta}\cdot\{\frac{w+M}{E+M}\cdot F_1(t) + [w + \frac{t}{4(E+M)}]\cdot\frac{F_2(t)}{M}\} \tag{8}$$

for the interactions of (respectively) $K^\pm$ with nucleons, while for K_S^0 regeneration one has to change the sign of the isovector part $F_K^V(t)$ of the kaon form factor $F_K(t) = \frac{1}{2}[F_K^S(t) \pm F_K^V(t)]$, the plus sign holding for charged kaons, the minus for the neutral ones. Here $\gamma = \alpha\cdot(w^2 - M^2 - m^2)/2qw$ and the Coulomb phase ϕ_C is defined as

$$\phi_C = -\gamma\log(\sin^2\frac{1}{2}\theta) + \gamma\cdot\int_{-4q^2}^{0}\frac{dt}{t}\cdot[1 - F_K(t)F_1(t)] \tag{9}$$

for charged kaons scattering on protons, while it reduces to

$$\phi_C = -\gamma \int_{-4q^2}^{0} \frac{dt}{t} F_K(t) F_1(t) \tag{9'}$$

for processes involving K^0's and/or neutrons.

We have denoted with w and θ respectively the total energy and the scattering angle in the c.m. frame, $q = [\frac{1}{4}w^2 - \frac{1}{2}(M^2 + m^2) + (M^2 - m^2)^2/4w^2]^{1/2}$ is the c.m. momentum (in the initial state: for inelastic processes, including charge exchange, we shall indicate final–state kinematical quantities with primes), E the total energy of the baryon in the c.m. frame, $E = (w^2 + M^2 - m^2)/2w$, and t the squared momentum–transfer, $t = M^2 + M'^2 - 2EE' + 2qq' \cos\theta$. We shall also use the laboratory–frame, initial–meson momentum $k = \frac{1}{2}(\omega^2 - m^2)^{1/2}$ and energy ω, related to the c.m. total energy via $\omega = (w^2 - M^2 - m^2)/2M$, and, besides t, the two other Mandelstam variables $s = w^2$ and u, the square of the c.m. total energy for the crossed channel $\bar{a}_2(0^-) + B_1(\frac{1}{2}^+) \to \bar{a}_1(0^-) + B_2(\frac{1}{2}^+)$, obeying *on the mass shell* the indentity $s + t + u = M^2 + M'^2 + m^2 + m'^2$.

In terms of the amplitudes G and H the c.m. differential cross sections for an unpolarized target (surely the case for experiments to be carried on at DAΦNE) take the simple form

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\alpha,\beta} |T_{\alpha\beta}|^2 = |G|^2 + |H|^2 \ . \tag{10}$$

The other observables possibly accessible at DAΦNE, in the strangeness–exchange processes $\overline{K}N \to \pi\Lambda$ and $\overline{K}N \to \pi\Sigma$, are the polarizations P_Y ($Y = \Lambda$ or Σ) of the final hyperons, measurable through the asymmetries α of their weak nonleptonic decays $\Lambda \to \pi^- p$ or $\pi^0 n$, for both of which we have an asymmetry $\alpha \simeq +0.64$, and $\Sigma^+ \to \pi^0 p$ for which the asymmetry is $\alpha \simeq -0.98$, while there is very little chance to be able to use the neutron decay modes $\Sigma^\pm \to \pi^\pm n$, which have the very small asymmetries $\alpha \simeq \pm 0.068$; we have for these quantities

$$P_Y \cdot (\frac{d\sigma}{d\Omega}) = 2 \ \mathrm{Im} \ (GH^*) \ . \tag{11}$$

Note that, for an $(S+P)$–wave parametrization (fully adequate at such low momenta), while the *integrated* cross sections depend only *quadratically* on the P–waves, both the first Legendre coefficients of the differential cross sections

$$L_1 = \frac{1}{2} \int_{-1}^{+1} \cos\theta \ (\frac{d\sigma}{d\Omega}) \ d\cos\theta = \frac{2}{3} \ \mathrm{Re} \ [T_{0+}(2T_{1+} + T_{1-})^* + \ldots] \tag{12}$$

and the polarizations

$$P_Y \cdot (\frac{d\sigma}{d\Omega}) = 2 \text{ Im } [T_{0+}(T_{1+} - T_{1-})^* + 3T_{1-}T_{1+}^* \cos\theta + \ldots] \sin\theta \tag{13}$$

are essentially *linear* in the small P–wave contributions, and give two complementary pieces of information on these latter. It is perhaps not useless to remind the reader that the low statistics of the experiments, performed *only* up to the late seventies, have not been enough to determine any of these parameters, yielding only very inaccurate (and utterly useless) determinations[2] of L_1 for the charged–hyperon production channels.

We shall now devote the last part of this section to show explicitly why this absence of *direct* information on the low–energy behaviour of the P–waves has been a serious shortcoming for $\overline{K}N$ amplitude analyses. Remember that we know, from *production* experiments, that the $I = 1$, $S = -1$ T_{1+} partial wave resonates *below* threshold at a c.m. energy around $w = 1384\ MeV$, the mass of the neutral, isovector member of the $J^P = \frac{3}{2}^+$ decuplet[5], but beyond this piece of information P–waves are practically unmeasured up to momenta above $\simeq 500\ MeV/c$.

One has to turn from the Pauli amplitudes G and H to the invariant amplitudes $A(s,t)$ and $B(s,t)$, defined in term of four–component Dirac spinors as

$$2\pi w\ T_{\alpha\beta} = \bar{u}_\alpha(p')[A(s,t) + B(s,t) \cdot \gamma^\mu Q_\mu] u_\beta(p)\ , \tag{14}$$

where $Q = \frac{1}{2}(q + q')$, the average between incoming– and outgoing–meson c.m. four–momenta: these amplitudes obey simple crossing relations and are free of kinematical singularities, so that they are the ones to be used, rather than G and H, for any analytical extrapolation purpose; it is also customary to use the combination $D(\nu,t) = A(\nu,t) + \nu \cdot B(\nu,t)$, where $\nu = (s-u)/2(MM')^{1/2}$, which has the same properties as $A(\nu,t)$ under crossing, and furthermore, for elastic scattering, obeys the optical theorem in the simple form

$$\text{Im } D(\nu, t=0) = k \cdot \sigma_{tot}\ , \tag{15}$$

where of course all electromagnetic effects must be subtracted on *both* sides.

One can rewrite A and B in terms of G and H, and thus reexpress them through the partial waves $T_{\ell\pm}$, by projecting eq. (14) on the different spin states (polarized *perpendicularly* to the scattering plane) and obtain in the most general kinematics

$$A(\nu,t) = \frac{4\pi}{(E+M)^{1/2}(E'+M')^{1/2}}\{[w+\frac{1}{2}(M+M')]G(w,\theta)+$$
$$+[(E+M)(E'+M')\{w-\frac{1}{2}(M+M')\}+\{\frac{1}{2}t+EE'-\frac{1}{2}(M^2+M'^2)\}\{w+\frac{1}{2}(M+M')\}]\cdot$$
$$\cdot\frac{H(w,\theta)}{qq'\sin\theta}\} \ , \tag{16}$$

and

$$B(\nu,t) = \frac{4\pi}{(E+M)^{1/2}(E'+M')^{1/2}}\{G(w,\theta)-$$
$$-[(E+M)(E'+M')-\frac{1}{2}t-EE'+\frac{1}{2}(M^2+M'^2)]\frac{H(w,\theta)}{qq'\sin\theta}\} \ . \tag{17}$$

Considering for sake of simplicity forward elastic scattering only, the amplitudes become, leaving out $D-$ and higher waves,

$$D(\nu,0) = \frac{4\pi w}{M}[T_{0+}+2T_{1+}+T_{1-}+\ldots] \tag{18}$$

and

$$B(\nu,0) = \frac{4\pi w}{Mq^2}[(E-M)T_{0+}-2(2M-E)T_{1+}+(E+M)T_{1-}+\ldots] \ ; \tag{19}$$

introducing the (complex) scattering lengths $a_{\ell\pm}$ and (complex) effective ranges $r_{\ell\pm}$ one can expand up to $O(q^2)$ the partial waves close to threshold, and obtain for the forward D amplitudes

$$D(q,0) = 4\pi(1+\frac{m}{M})\{a_{0+}+ia_{0+}^2q+[2a_{1+}+a_{1-}-(a_{0+}+\frac{1}{2}r_{0+})a_{0+}^2-\frac{a_{0+}}{2Mm}]q^2+\ldots\} \ , \tag{20}$$

dominated by the S–waves, while for the B amplitudes the same approximations give

$$B(q,0) = \frac{2\pi}{M}(1+\frac{m}{M})[a_{0+}-4M^2(a_{1+}-a_{1-})+ia_{0+}^2q+\ldots] \ , \tag{21}$$

where the factor $4M^2 \simeq 90 \ fm^{-2}$ enhances considerably the contributions by the low–energy P–waves (virtually unkown), rendering practically useless the unsubtracted dispersion relation for the better converging B amplitudes, so important for the πN case in fixing accurately the values of the coupling constant f^2 and of the S–wave scattering lengths[6].

3. The Coulomb corrections and the kaonic hydrogen "puzzle".

The Coulomb shifts $\sigma_{\ell\pm}$ can be separated into the real, purely Coulomb phases σ_ℓ and the *complex* Coulomb corrections $\Delta_{\ell\pm}$,

$$\sigma_{\ell\pm} = \sigma_\ell + \Delta_{\ell\pm} \ , \tag{22}$$

where $\sigma_\ell = \arg\Gamma(\ell + 1 + i\gamma)$ to lowest order in α, and the Coulomb corrections $\Delta_{\ell\pm}$ have to be explicitly computed from a first–order "ansatz" on the purely nuclear interaction. They have been computed in a dispersion–relation formalism by Tromborg, Waldenstrøm and Øverbø[8] for the πN case: the same formalism[7] could in principle be extended (but has not been up to now) also to the KN and $\overline{K}N$ ones.

Since this method (as the other methods as well) uses a "reference" strong interaction, minor corrections remain to be applied to the *observed* phases $\delta_{\ell\pm}$ and elasticities $\eta_{\ell\pm}$, to extract their *purely nuclear* parts[6]; a way of removing them efficiently has been devised by the Karlsruhe–Helsinki group: it consists in starting from a (preliminary) set of phase shifts, (i) calculating from them the corrections in the above–mentioned dispersive formalism[7,8], then (ii) the changes in the observables brought about by these latter, and finally (iii) correcting the data for these effects and (iv) starting the phase–shift analysis all over again, this time from the "corrected data". This procedure has turned out to be both self–consistent and fast[6].

Of course, the accuracy of a dispersive approach is limited by the *overall* accuracy of the experimental data, from which the "reference" interaction is extracted, and particularly by that of the low–energy ones, which in the $\overline{K}N$ interaction is dominated by the very low statistics of the most accurate – and recent – experiment[2].

As an alternative to the above dispersive approach[7,8], one can describe the scattering and production processes by either a Schrödinger[9,10] or a Klein–Gordon[11,12] multichannel wave equation, with both Coulomb and strong–interaction potentials; such formalisms have the advantage, when continuing the c.m. momentum q of the K^-p system from the real to the imaginary axis, of predicting at the same time the hadronic level shifts and widths for the kaonic–hydrogen atomic levels, present as an infinite number of poles just below the elastic threshold in every partial wave, having the threshold as an accumulation point.

Such an approach is preferable when working at very low momenta, in absence of previous experimental information of comparable quality, which is the case for the foreseeable KN and $\overline{K}N$ experiments to be carried out at DAΦNE. We shall in the following discuss in detail only the relatively less know Klein–Gordon formalism, leaving out the Schrödinger one, whose details are easily workable out, following e.g. the papers listed in reference (10) as a guideline.

The advantage of the Klein–Gordon formaliam over the Schrödinger one lies mainly

in the fact that the fine structure of mesic atoms is clearly (and directly) produced by the former but not by the latter: one would thus be tempted to use it also to calculate the Coulomb modifications to the scattering amplitudes, particularly in the low–energy region considered here, where both formalisms give reasonable approximations to the true kinematics of the two–particle system. We have indeed, for the c.m. kinetic energy $T_{c.m.} = [M^2+q^2]^{\frac{1}{2}}+[m^2+q^2]^{\frac{1}{2}}-(M+m)$, the approximations $T_S = q^2/2\mu$ – with the reduced mass $\mu = Mm/(M+m)$ – for the Schrödinger equation, and $T_{KG} = E_{KG} - \mu = [\mu^2+q^2]^{\frac{1}{2}} - \mu$ for the Klein–Gordon one, both differing from each other and from $T_{c.m.}$ at $\mathrm{O}(q^4)$ only.

However, to include an interaction with all the good symmetry properties expected for pseudoscalar mesons interacting with a baryonic "source", a Klein–Gordon equation has to possess at least a four–vector and a scalar term (being concerned mostly with the S–waves, we shall neglect for the moment the tensor part of the interaction), of the form (in the "static limit")

$$\{\nabla^2 - \mu^2 - 2\mu\ S(r) - [E_{KG} + U(r) - e\varphi_C(r)]^2\}\ \Psi(\mathbf{r})\ =\ 0\ , \tag{23}$$

where the *effective* potentials $S(r)$ and $U(r)$ (real in the elastic, single–channel case) are respectively even and odd under C–parity conjugation of the meson fields (in a multi–channel case, such as the one we are studying, $S(r)$ and $U(r)$ will be real, nondiagonal matrices, and E_{KG} and μ diagonal ones, of dimension 6×6 for K^-p and 5×5 for $\overline{K}^0 p$, and the product $2\mu S(r)$ will have to be replaced by the anticommutator $\{\mu, S(r)\}$): they can be separated only if data on *both* the s– and u–channel reactions are available *at the same c.m. energy*, which is clearly *not* the case for the pion–hyperon channels, since the processes $\pi p \to KY$ can be accessed only at c.m. energies above $m_K + M_Y$, well above the $\overline{K}N$ c.m. energy available at DAΦNE, below 1442 MeV for K^-p (and of only 1444 MeV for $K^0_L p$).

One has therefore to abandon the real potentials for complex ones, reducing oneself to treat with the Klein–Gordon equation (23) only the two, coupled K^-p and $\overline{K}^0 n$ channels: however, one ambiguity is still left, in the separation of the absorption effects from the neglected $\pi\Lambda$ and $\pi\Sigma$ channels between the two potentials $S(r)$ and $U(r)$. This ambiguity could be resolved[12], under the *reasonable* assumption of an energy dependence of the potentials[13] as gentle as expected from forward dispersion relations and the absence of any dynamical effect apart from the $\Lambda(1405)$ S–wave resonance below threshold[4,5], if the energy shift and width of the ground state for the K^-p atomic system had been actually measured: the three experiments performed in the late seventies and early eighties

on this system[14,15] have been widely quoted as to report *attractive* hadronic shifts (and small widths), contrary to all *reasonable* expectations[16,17], but in fact none of the three has produced convincing evidence for the identification of the X–ray transitions they claim having observed as lines belonging to the K–series of the K^-p atomic system. The only experiment having an acceptable signal–to–background ratio (the first one conducted by Davies *et al.*[14] in the same beam–line of the TST Collaboration experiment[2] at NIMROD), observed only *one* clear transition, which could well have been the K^-p K_β line and *not* the K_α they claimed it to be (see the comments about this problem in both reviews listed in reference (16)). For the interpretation of the K^-p X–ray lines the evaluation of their intensities is essential: a recent cascade calculation has been performed by Reifenröther and Klempt[18]. A new experiment, which expects to achieve a much better signal–to–background ratio, is presently under way at KEK[19], and studies on the possibility of a kaonic hydrogen experiment at DAΦNE must be seriously considered, for this measurement will be complementary to, if not substitutive of, a conventional beam–target experiment such as the KEK one, due to the completely different nature in the possible X-ray backgrounds.

The above ambiguity is not as serious an obstacle as it might appear at first glance: the "strong potentials" $S(r)$ and $U(r)$ are usually assumed to have a simple, predetermined r–dependence (with Rasche and Woolcock[10] we prefer square wells[12], $S(r) = S_0\ \Theta(r_0 - r)$ and $U(r) = U_0\ \Theta(r_0 - r)$, since these yield very simple, analytic "inner" and "outer" solutions, i.e. respectively Bessel and Kummer (or Whittaker) functions, once $\varphi_C(r)$ is substituted by its average value inside the square well, and vacuum polarization is neglected at this stage, and later treated as a small perturbation, though this requires some particular care[20] in normalising the Klein–Gordon wavefunctions, with respect to the Schrödinger ones), and the ambiguity reduces then to a *single* free parameter, e.g. the ratio $1 \geq \rho = \mathrm{Im}S_0/(\mathrm{Im}S_0 + \mathrm{Im}U_0) \geq 0$ (since unitarity dictates both kind of potentials, when absorptive, to have a *negative* imaginary part). The calculation can now go on along the same lines as the Helsinki–Karlsruhe one (*modulo* the different algorithms): one can start from an "initial" set of phases, (i) turn them into a pair of potentials S, U, (ii) compute the new corrections, (iii) use these latter to extract new phases, and (iv) start again from the point (i) till input and output differ by less than a small, pre–fixed amount.

The procedure relies, rather than on cumbersome integrations (as in the dispersive approach), on well known analytic functions, most of which if not all already available

in software packages, easy to adapt to the case under study; the same software, after extrapolation of the potentials to threshold through a reasonably small correction (q in the region covered by DAΦNE's K^-'s goes from about 59 to about 78 MeV/c only), since it contains automatically the Deser–Goldberger–Baumann–Thirring formula for the hadronic shifts and widths[12,21], can also produce, *mutatis mutandis*, accurate predictions for shifts and widths of the K^-p $1s$ (and Np, once the P–waves are determined as well) atomic states, and therefore for the *whole* K–series, for any chosen set of the only two free parameters left, i.e. the ratio ρ and the square–well radius r_0, to be varied at most from once to twice the r.m.s. charge radius of the proton (or up and down a 30% around 1 fm).

4. Open channels and baryon spectroscopy at DAΦNE.

As mentioned above, in the momentum region which could be explored by the kaons coming from the decays of a ϕ–resonance formed at rest in an e^+e^- collision, we have only data from low–statistics experiments, mostly hydrogen bubble–chamber ones on K^-p (and K^-d) interactions[1,2] (dating from the early sixties trough the late seventies), plus scant data from K_L^0 interactions and K_S^0 regeneration, mostly on hydrogen[3].

The channels, open at a laboratory energy $\omega = \frac{1}{2}m_\phi$ (for $K^\pm$'s to obtain the exact value of ω one has to include their energy losses through ionization as well), are tabulated below for interactions with *free* protons and neutrons, together with their threshold energies w_{thr} (in MeV), strangeness and isospin(s). We do not list K^+–initiated processes, which are (apart from charge exchange) purely elastic in this energy region.

For interactions in hydrogen, the c.m. energy available for each final state is limited by momentum conservation to the initial total c.m. energy, equal (neglecting energy losses) to $w = (M_p^2 + m_K^2 + M_p m_\phi)^{1/2}$, or 1442.4 MeV for incident $K^\pm$'s and 1443.8 MeV for incident K_L^0's. Energy losses for charged kaons can be exploited (using the inner parts of the detector as a "moderator") to explore K^-p interactions in a *limited* momentum range, possibly down to and below the charge–exchange threshold at $w = 1437.2$ MeV, corresponding to a K^- laboratory momentum of about 90 MeV/c.

For interactions in deuterium (or in heavier nuclei), momentum can be carried away by "spectator" nucleons, and one can explore each inelastic channel from the highest available energy down to threshold. The possibility of reaching energies below the $\overline{K}N$ threshold is particularly desirable, since the $\overline{K}N$ unphysical region contains two resonances[4,5], the

$I = 0$, S–wave $\Lambda(1405)$ and the $I = 1$, $J^P = \frac{3}{2}^+$ P–wave $\Sigma(1385)$, observed mostly in production experiments (and, in the first case, with very limited statistics[22]), so that the information on their couplings to the $\overline{K}N$ channel relies *entirely* on extrapolations below threshold of the analyses of the low–energy data. The coupling of the $\Sigma(1385)$ to the $\overline{K}N$ channel, for instance, can at present be determined only via forward dispersion relations involving the *total sum* of data collected at $t \simeq 0$, but with uncertainties which are, *at their best*, still of the order of 50% of the value expected from flavour–$SU(3)$ symmetry[23]; as for the $\Lambda(1405)$, even its spectroscopic classification is still an open problem, *vis–à–vis* the paucity and (lack of) quality of the *best available* data[4,24].

Table I

Channel	w_{thr}/MeV	S	I
$K^-p, K^0_Ln \;\rightarrow\; \pi^0\Lambda$	1250.6	–1	1
$K^-p, K^0_Ln \;\rightarrow\; \pi^0\Sigma^0$	1327.5	–1	0
$K^-p, K^0_Ln \;\rightarrow\; \pi^-\Sigma^+$	1328.9	–1	0,1
$K^-p, K^0_Ln \;\rightarrow\; \pi^+\Sigma^-$	1337.0	–1	0,1
$K^-p, K^0_Ln \;\rightarrow\; \pi^0\pi^0\Lambda$	1385.6	–1	0
$K^-p, K^0_Ln \;\rightarrow\; \pi^+\pi^-\Lambda$	1394.8	–1	0,1
$K^-p, K^0_Ln \;\rightarrow\; K^-p$	1431.9	–1	0,1
$K^-p, K^0_Ln \;\rightarrow\; K^0_Sn$	1437.2	–1	0,1
” ”	”	+1	1†

†) This amplitude only appears in the regeneration process $K^0_Ln \rightarrow K^0_Sn$.

Table II

Channel	w_{thr}/MeV	S	I
$K^-n \;\rightarrow\; \pi^-\Lambda$	1255.2	–1	1
$K^-n \;\rightarrow\; \pi^-\Sigma^0$	1332.1	–1	1
$K^-n \;\rightarrow\; \pi^0\Sigma^-$	1332.1	–1	1
$K^-n \;\rightarrow\; \pi^0\pi^-\Lambda$	1388.2	–1	1
$K^-n \;\rightarrow\; K^-n$	1433.2	–1	1

Table III

Channel	w_{thr}/MeV	S	I
$K_L^0 p \;\rightarrow\; \pi^+\Lambda$	1255.2	–1	1
$K_L^0 p \;\rightarrow\; \pi^0\Sigma^+$	1324.3	–1	1
$K_L^0 p \;\rightarrow\; \pi^+\Sigma^0$	1332.1	–1	1
$K_L^0 p \;\rightarrow\; \pi^0\pi^+\Lambda$	1388.2	–1	1
$K_L^0 p \;\rightarrow\; K^+ n$	1433.2	+1	0,1
$K_L^0 p \;\rightarrow\; K_S^0 p$	1435.9	+1	0,1
" "	"	–1	1

A *formation* experiment on *bound* nucleons in an (almost) 4π apparatus with good efficiency and resolution for low–momentum γ's (such as KLOE) can reconstruct and measure a channel such as $K^-p \rightarrow \pi^0\Sigma^0$ (only *above* the $\overline{K}N$ threshold) or $K^-d \rightarrow \pi^0\Sigma^0 n_s$ (both *above* and *below* threshold), which is pure $I = 0$: up to now all analyses on the $\Lambda(1405)$ have been limited to charged channels[22], being thus forced to assume the $I = 1$ contamination in their samples to be either negligible or smooth and not interfering with the resonance signal (remember that it is common knowledge[5] that there is a P–wave resonance in the $I = 1$, $\pi^\pm\Sigma^\mp$ channels at 1384 MeV!). This situation is particularly unsatisfactory, in view of the fact that the various spectroscopic models proposed for the classification of the $\Lambda(1405)$ differ mostly in the detailed resonance *shape*, rather than in its couplings[24]: now, it is precisely the shape which could be drastically changed even by a moderate amount of interference with an $I = 1$ "background". Note also that, having in the same apparatus, and at almost the same energy *tagged* K^- and K_L^0 produced at the same point, one can separate $I = 0$ and $I = 1$ channels with a minimum of systematic uncertainties, by measuring all channels $K_L^0 p \rightarrow \pi^0\Sigma^+$, $\pi^+\Sigma^0$ and $K^-p \rightarrow \pi^-\Sigma^+$, $\pi^+\Sigma^-$, besides, of course, the above–mentioned, pure $I = 0$, $K^-p \rightarrow \pi^0\Sigma^0$ one.

Another class of inelastic processes, which are expected to be produced (even if at a much smaller rate) by DAΦNE's $\overline{K}$'s, is radiative capture, leading in hydrogen to the final states $\gamma\Lambda$ and $\gamma\Sigma^0$ for incident K^-'s, and, for incident K_L^0's, to the final state $\gamma\Sigma^+$: in deuterium, one expects to observe the capture processes by neutrons, $K^-d \rightarrow \gamma\Sigma^- p_s$ and $K_L d \rightarrow \gamma\Sigma^0 p_s$, $\gamma\Lambda p_s$ as well. Observation of these processes has been limited up to now to searches for photons emitted after capture of K^-'s stopped in liquid hydrogen (and deuterium): but in these experiments the spectra are dominated by photons from

unreconstructed π^0 and Σ^0 decays[25]. This poses serious difficulties already at the level of separation of signals from background, since (in K^-p capture at rest) only the photon line from the $\gamma\Lambda$ final state falls *just above* the endpoint of the photons from decays of the π^0's in the $\pi^0\Lambda$ final state, while that from $\gamma\Sigma^0$ falls right on top of this latter: indeed, these experiments were able to produce, within quite large errors, only an estimate of the respective branching ratios.

The 4π geometry possible at DAΦNE, combined with the "transparency" of a KLOE–like apparatus, its high efficiency for photon detection and its good resolution for spatial reconstruction of the events, should make possible the full identification of the final states and therefore the measurement of the absolute cross sections for these processes, although in flight and not at rest.

This difference can be appreciated when comparing with theoretical predictions: the main contributions to radiative captures are commonly thought to come from radiative decays of resonant levels in the $\overline{K}N$ system[26], while the total hyperon production cross section is expected to come from both resonant and non–resonant intermediate states. An estimate of the branching ratios would therefore be quite sensitive to the latter, while a prediction of the absolute cross sections should not.

Data[25] are presently indicating branching ratios around 0.9×10^{-3} for $K^-p \to \gamma\Lambda$ and 1.4×10^{-3} for $K^-p \to \gamma\Sigma^0$, with errors of the order of 15% on both rates: most theoretical models[27] tend to give the first rate larger than the second, with both values consistently higher than the observed ones. Only a cloudy–bag–model estimate[28] exhibits the trend appearing (although only at a 2σ level, and therefore waiting for confirmation by better data) from the first experimental determinations, but this is the only respect in which this model agrees with the data, still giving branching ratios larger than observations by a factor two[29].

Data are also interpretable in terms of $\Lambda(1405)$ electromagnetic transition moments[26]: this interpretation of measurements taken at a single energy, or over a limited interval, is clearly subject to the effect of the interference between this state and all other contributions, such as the Λ– and Σ–hyperon poles and other resonant states such as the $\Sigma(1385)$ and the $\Lambda(1520)$, not to mention t–channel exchanges (since at least K–exchange has to be included, to ensure gauge invariance of the Born approximation). An extraction of the $\Lambda(1405)$ moments, relatively freer of these uncertainties, requires measurements of the final states $\gamma\Lambda$ and $\gamma\Sigma$ (if possible, in different charge states) over the *unphysical region*,

using (gaseous) deuterium or helium as a "target". Rates are expected to be only of the order of 10^4 $events/y$, but it must be kept in mind that such a low rate (by DAΦNE's standards) corresponds already to statistics *two* orders of magnitude above those of the *best* experiment performed till now on the shape of the $\Lambda(1405) \to \pi\Sigma$ decay spectrum[22].

5. The K-matrix (or M-matrix) formalism.

An adequate description of the low–energy $\overline{K}N$ partial waves must couple at least the dominant, two–body inelastic channels to each other and to the elastic one; the three–body channel $\pi\pi\Lambda$ is expected to be suppressed, for $J^P = \frac{1}{2}^-$, by the angular momentum barrier, but it could contribute appreciably to the $I = 0$, $J^P = \frac{1}{2}^+$ P–wave, due to the strong final–state interaction of two pions in an $I = 0$ S–wave. Note that *most* bubble chamber experiments were unable to fully reconstruct the events at the lowest momenta, and therefore often assumed all *directly* produced Λ's to come from the $\pi\Lambda$ channel *alone*, neglecting altogether the small $\pi\pi\Lambda$ contribution.

The appropriate formalism is to introduce a K–matrix description (sometimes it is convenient to use, instead of the K–matrix, its inverse, also known as the M–matrix), defined in the isospin eigenchannel notation as

$$\mathbf{K}_{\ell\pm}^{-1} = \mathbf{M}_{\ell\pm} = \mathbf{T}_{\ell\pm}^{-1} + i\ \mathbf{Q}^{2\ell+1} \ , \tag{24}$$

for both $I = 0, 1$ S–waves (and perhaps also for the four P–waves as well, or at least for the $I = 1$, T_1+ wave, which resonates below threshold). The K–matrices, assuming $SU(2)$ symmetry, describe the S–wave data at a given energy in terms of *nine* real parameters (six for $I = 1$ and three for $I = 0$), while the experimentally accessible processes can be described, assuming pure S–waves in the same symmetry limit, by only *six* independent parameters, which can be chosen to be the two (complex) amplitudes A_0, A_1 for the $\overline{K}N \to \overline{K}N$ channel, the phase difference ϕ between the $I = 0$ and $I = 1$ $\pi\Sigma$ production amplitudes, and the ratio ϵ between the $\pi\Lambda$ production cross section and that for total hyperon production in an $I = 1$ state[30].

Thus a single–energy measurement does not allow a complete determination of the K–matrix elements at that energy. Using high–statistics measurements at different momenta, and assuming either *constant* K–matrices or (if more complexity were needed) *effective–range* M–matrices could *in principle* fully determine the matrix elements: but for this to

be possible one has to be able to subtract out the (small) P–wave contributions to the integrated cross sections

$$\sigma = 4\pi L_0 = 2\pi \int_{-1}^{+1} (\frac{d\sigma}{d\Omega})\ d\cos\theta = 4\pi[|T_{0+}|^2 + 2|T_{1+}|^2 + |T_{1-}|^2 + \ldots]\ , \qquad (25)$$

which could be obtained either from L_1 alone, for the elastic and charge–exchange channels, or from both L_1 and P_Y, which give *complementary* information, for the hyperon production channels. None of these quantities has been measured with the desirable accuracy up to now: the TST Collaboration tried to extract L_1 from some of their low–statistics data, and found results consistent with the tail of the $\Sigma(1385)$ resonance in the $I = 1$, T_{1+} wave, but also consistent (at the 2σ level) with zero within their obviously very large errors[2]. At the same level of accuracy, one should also be able to isolate and separate out the $\pi\pi\Lambda$ channel contribution as well.

Remember that an accurate analysis has also to include the *complete* e.m. corrections: up to now all $\overline{K}N$ analyses have relied on the formula derived by Dalitz and Tuan[31] for a pure S–wave scattering with a weak hadronic, short–range interaction, which is hardly the case for the $\overline{K}N$ system around threshold.

To fix the *redundant* K–matrix parameters, different authors have tried different methods: some have used the data on the shape of the $\pi\Sigma$ spectrum from production experiments[32], others have constrained the amplitudes in the unphysical region by imposing consistency with dispersion relations for the amplitudes D for both $K^{\pm}p$ and $K^{\pm}n$ forward elastic scattering[33,34], relying on the existence of accurate data on the total cross sections at higher energies. More recently, some attempts have been made to combine both constraints into a "global" analysis, but with no better results than each of them taken separately[35].

Unfortunately, neither of these methods has been very powerful, because of the low statistics of the $\pi\Sigma$ production data on one side, and on the other because of the need to use for the dispersion relations the often not very accurate information (and particularly so for the $K^{\pm}n$ amplitudes) on the real–to–imaginary–part ratios.

We list below (without errors, often meaningless since the parameters are strongly correlated, and therefore not even quoted by some of the authors) the constant K–matrices found by Chao *et al.* using the first method[32] (which *did not* include the TST Collaboration data), and the more complex parametrization found by A.D. Martin using the second[34] (and including the *preliminary* TST data). Note that to describe the data for $I = 0$ both

above and *below* threshold A.D. Martin was forced to introduce a "constant–effective–range" M–matrix, where $\mathbf{M}^{(0)} = (\mathbf{K}^{(0)})^{-1} = \mathbf{A} + \mathbf{R}q^2$, with three more "effective range" parameters, so that to make the two analyses comparable we list separately his *threshold* K–matrix values.

The purpose of this table is to show that there is considerable uncertainty even on the value of the $K^{(I)}_{NN}$ elements of the K–matrices (the real parts of the corresponding scattering lengths): the data have been re–analyzed by Dalitz *et al.*[35], using *both* sets of constraints with different weigths and different parametrizations, and yielding a variety of fits, all of them of about the same overall quality and none of them improving very much over the above ones.

Just to highlight the difficulties met in describing the data (probably plagued by inconsistencies between different experiments, and by large systematic uncertainties), we point out that A.D. Martin himself[34] found that including in his analysis a $\Sigma(1385)$ resonance at the right position, with the width given by the production experiments (and listed in the Particle Data Group tables[5]) and the coupling to the $\overline{K}N$ channel dictated by flavour–$SU(3)$ symmetry, was worsening rather than improving the fits obtained *neglecting* it altogether: his analysis therefore considers the Σ Born–term contribution a "superposition" of the former and of that of the P–wave resonance, a rather unsavoury situation considering the different J^P quantum numbers of the two states, which may raise questions about the applicability of his analysis away from $t \simeq 0$. Note that a similar superposition has to be considered in the $K^{\pm}p$ dispersion relations for the Σ– and Λ–pole contributions, which can not be separated from each other due to their being very close in the ν–variable plane: here however the two states contribute to the same partial wave, and the Σ–pole can be independently extracted from $K^{\pm}n$ scattering (or K^0_S regeneration on protons) data[36].

In the analysis of the low–energy data collected in the past on these processes, one of the main difficulties comes from the large spread in momentum of the typical low–energy kaon beams, for $K^{\pm}$'s because of the degrading in a "moderator" of the higher–energy beams needed to transport the kaons away from their production target, for K^0_L's because of the large apertures needed to achieve satisfactory rates in the targets (typically bubble chambers): this made unrealistic the proposals (advanced from the early seventies) of better determining the low–energy K–matrices by studying the behaviour of the cross sections for K^-p–initiated processes at the $\overline{K}^0 n$ charge–exchange threshold[37]. The high momentum resolution available at DAΦNE will instead make such a goal a realistically

achievable one.

Table IV

Chao et al.		*A. D. Martin*	
$K^{(0)}_{NN} = -1.56 fm$	$A_{NN} = -0.07 fm^{-1}$	$R_{NN} = +0.18 fm$	$K^{(0)}_{NN}(0) = -1.65 fm$
$K^{(0)}_{N\Sigma} = -0.92 fm$	$A_{N\Sigma} = -1.02 fm^{-1}$	$R_{N\Sigma} = +0.19 fm$	$K^{(0)}_{N\Sigma}(0) = -0.87 fm$
$K^{(0)}_{\Sigma\Sigma} = +0.07 fm$	$A_{\Sigma\Sigma} = +1.94 fm^{-1}$	$R_{\Sigma\Sigma} = -1.09 fm$	$K^{(0)}_{\Sigma\Sigma}(0) = +0.06 fm$
$K^{(1)}_{NN} = +0.76 fm$			$K^{(1)}_{NN} = +1.07 fm$
$K^{(1)}_{N\Sigma} = -0.97 fm$			$K^{(1)}_{N\Sigma} = -1.32 fm$
$K^{(1)}_{N\Lambda} = -0.66 fm$			$K^{(1)}_{N\Lambda} = -0.30 fm$
$K^{(1)}_{\Sigma\Sigma} = +0.86 fm$			$K^{(1)}_{\Sigma\Sigma} = +0.27 fm$
$K^{(1)}_{\Sigma\Lambda} = +0.51 fm$			$K^{(1)}_{\Sigma\Lambda} = +1.54 fm$
$K^{(1)}_{\Lambda\Lambda} = +0.04 fm$			$K^{(1)}_{\Lambda\Lambda} = -1.02 fm$

In this case one can no longer assume $SU(2)$ to be a good symmetry of the amplitudes: under the (reasonable) assumption that the forces are still $SU(2)$–symmetric, one can however still retain the previous K–matrix formalism, but one can no longer decouple the different isospin eigenchannels[10]. Introducing the orthogonal matrix $\mathbf{R}$, which transforms the six isospin eigenchannels for $\overline{K}N$ ($I = 0, 1$), $\pi\Lambda$ ($I = 1$ only) and $\pi\Sigma$ ($I = 0, 1, 2$) into the six physical charge channels K^-p, $\overline{K}^0 n$, $\pi^0\Lambda$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$, and the diagonal matrix $\mathbf{Q}_c$ of the c.m. momenta for these latter, one can rewrite the T–matrix for the S–waves in the isospin–eigenchannel space as

$$\mathbf{T}_I^{-1} = \mathbf{K}_I^{-1} - i\mathbf{R}^{-1}\mathbf{Q}_c\mathbf{R} \ , \tag{26}$$

where $\mathbf{K}_I$ is a box matrix with zero elements between channels of different isospin, and $\mathbf{R}^{-1}$ $\mathbf{Q}_c$ $\mathbf{R}$ is of course no longer diagonal.

Apparently this involves one more parameter, since it also contains the element $K^{(2)}_{\Sigma\Sigma}$: in practice, if one is interested in the behaviour of the cross sections in the neighbourhood of the $\overline{K}N$ charge–exchange threshold, one can take the c.m. momenta in the three $\pi\Sigma$

channels as equal, so that the $I = 2$, $\pi\Sigma$ channel decouples from the $I = 0,\ 1$ ones, since the "rotated" matrix $\mathbf{R}^{-1}\mathbf{Q}_c\mathbf{R}$ has now only two non–zero, off-diagonal elements, equal to $\frac{1}{2}(q_0 - q_-)$ (where the subscripts refer to the kaon charges), between the $I = 0$ and $I = 1$ $\overline{K}N$ channels, the diagonal ones being almost the same as in the *fully* $SU(2)$–symmetric case, with only the substitution to the $\overline{K}N$ channel momentum q of the average over the two charge states, $\frac{1}{2}(q_0 + q_-)$. $K^{(2)}_{\Sigma\Sigma}$ would however be important for describing accurate experiments on $\pi\Sigma$ and $\pi\Lambda$ mass spectra in the unphysical region below the $\overline{K}N$ threshold without recourse to an $SU(2)$–symmetry limit: but the state–of–the–art of our understanding of wave–functions, even for the lightest nuclei, is not such as to make these isotopic–symmetry–breaking corrections relevant.

5. Low–energy K^+ scattering is important, too.

Better information on the $S = +1$ system is also essential in several cases. We limit ourselves to mention only two of the problems coming to our mind. Isospin symmetry, as can be seen from the previous section, is an essential ingredient in the phenomenological analysis of the KN system, apart from obvious mass–difference effects, apparent only in the close proximity of the thresholds, which one can describe by modifying the K–matrix formalism as outlined above[10].

One way to check isospin symmetry is to relate the amplitudes derived from *charged* kaon scattering to the data from K^0_S regeneration. Since isospin relates the scattering of charged kaons on protons to the regeneration on neutrons (and *vice versa*), the test is better performed on an isoscalar nuclear target, such as deuterium or ^{4}He. We should have indeed, apart from kinematical corrections and CP–violation effects,

$$T(K^0_L p \to K^0_S p) = \frac{1}{2}[T(K^0 p \to K^0 p) - T(\overline{K}^0 p \to \overline{K}^0 p)] =$$

$$= \frac{1}{2}[T(K^+ n \to K^+ n) - T(K^- n \to K^- n)] \tag{27}$$

and

$$T(K^0_L n \to K^0_S n) = \frac{1}{2}[T(K^0 n \to K^0 n) - T(\overline{K}^0 n \to \overline{K}^0 n)] =$$

$$= \frac{1}{2}[T(K^+ p \to K^+ p) - T(K^- p \to K^- p)]\ ; \tag{28}$$

when we introduce these equalities in a nuclear scattering calculation, as in *e.g.* a Glauber model, all *elastic* multiple scattering effects should apply equally to both the right– and

left–hand sides of the equalities for an isoscalar nucleus, protecting the identity from a large fraction of the "nuclear" effects[38].

Up to now such tests would have been possible only at higher momenta, where the opening of inelastic channels in the $S = +1$ systems complicates calculations further: a test performed in the elastic region of this system should make things simpler and clearer, at least in the $S = +1$ sector.

The second problem, related in many theoretical analyses to observations from inelastic electron and muon scattering on nuclei, namely to changes in the electromagnetic properties and in the deep–inelastic structure functions of nucleons bound in nuclei with respect to the free ones, is the "antishadowing" effect observed at momenta around 800 MeV/c for K^+ scattering on nuclear targets[39]. Conventional Glauber–model calculations[40] led to expect a ratio $(2\sigma_A)/(A\sigma_D)$ slightly less than unity and decreasing with both the kaon momentum and the target mass number A, while the measured values were larger than unity and increasing with momentum. This led to think, as an explanation of this and of the aforementioned electromagnetic phenomena, of a "swelling" of the bound nucleons with respect to free ones, in line with some of the explanations put forward for the "nuclear" EMC effect, though at a much higher energy scale[41].

New data have recently confirmed this trend[42], but only for momenta higher than approximately 600 MeV/c; a possibility coming to mind is that the opening of inelastic channels, such as πKN (or more simply quasi–two–body ones as KN^* and K^*N), might necessitate the introduction of inelastic intermediate states absent in a conventional Glauber–model calculation, phenomenon analogous to the need to introduce inelastic diffraction in the intermediate steps of a multiple–scattering formalism to explain diffractive processes on nuclei at much higher energies: thus the data would be just showing the opening of the threshold for such a phenomenon, particularly visible in the K^+–scattering case because of the extremely long mean–free path of this hadron in nuclear matter (about 7 fm).

Measurements of the K^+ cross sections on different nuclei in DAΦNE's kinematical region, where K^+N interactions are purely elastic, should help close the issue when compared with accurate Glauber–model calculations[40].

We would like to close reminding the reader that information on the $S = +1,\ I = 0$ channel in this energy region is coming *entirely* from extrapolations from higher–momentum data, since K^+–scattering (and regeneration) data on deuterium are *not* available at

momenta lower than about 300 MeV/c: at present we have only a generic idea about the order of magnitude of the *absolute value* of the KN $I=0$ scattering length, expected to be of the order of some times 10^{-2} fm from forward dispersion relations and the lowest–momentum regeneration data[33,34]. An accurate measurement of the cross sections for K^+ *incoherent* scattering on deuterium, possible at DAΦNE over a wide angular range, would thus give us the first *direct* measurement of this quantity.

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